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Entropy generation at the bow shock + Magnetic field Reconnection and Particle acceleration at neutral points

- In this last lecture, I will discuss two important problems: 1. Generation of *entropy in collisionless* shocks and 2. *particle acceleration* in magnetic neutral points.

- The best-known collisionless shock is Earth's bow shock. We will examine data from the bow shock and see what they can tell us about *entropy* generation in ollisionless plasmas.

- For particle acceleration in neutral points, we will discuss how the magnetic reconnection came to space physics and the recent work of Takeuchi (2002) who has solved the relativistic Lorentz equation in ***E**x**B*** geometry. He shows particles are accelerated in the ***E**x**B*** direction and the suggestion is that the solutions are important for space, solar and astrophysical plasmas.

Entropy in Collisionless Plasmas

- Entropy is generated in a *macroscopic system that are irreversible* when the underlying mechanics of *individual particles are reversible*.

- Ludwig Boltzmann developed the concept of entropy in an atomic model of gases to resolve the mystery of why macroscopic systems are irreversible while the mechanics of individual particles in the systems are reversible.

- Krall 91995) defined collisionless shock waves involve *irreversible processes* and represent transitions between two regions of *local thermodynamic equilibrium*. Entropy generation is thus expected.

Entropy

• Boltzmann's *entropy* is

$$S = -k_B H,$$

where

$$H = \int f \log f \, d^3v,$$

Here f is one particle distribution function.

• Differentiation of *H* leads to Boltzmann's *H-theorem* is

$$dH/dt = \int (1 + \log f) \, \partial f/\partial t \, d^3v \leq 0$$

Equality holds ***only*** if f is Maxwellian. $\partial H/\partial t$ is always ***negative*** and given a system can be in many different configurations, $\partial H/\partial t$ evolves to a state of maximum entropy

3D Plasma Instruments Can Measure f

1. How do we compute entropy from measured *f (v)*?

2. What Assumptions are made in the calculations?

3. Show an example of how entropy behaves across the bow shock

4. Show entropy production is tied to mechanisms that produce *non-thermal distributions*

5. Compare to Vlasov theory of how entropy flux should behave.

6. Questions that are not answered by our observations

Assumptions of Analysis

- Boltzmann's analysis considered a gas at rest that is changing in time.

- This situation is similar to an instrument co-moving within a magnetic flux tube that of steady state SW as the flux tube crosses the bow shock.

- SW is nearly two orders of magnitude faster than the spacecraft and the SC moves slowly with respect to the bow shock. Hence, we interpret observed time variations as due to SC motion through spatial structures.

- Consistent with this interpretation, we also assume measurements along the SC track as a history of the plasma volume that traveled the same track.

Measurement of Entropy

- Cluster and Double Star routinely measure *3D* $f(\mathbf{r}, \mathbf{v}, t)$ of SW in regions *upstream, downstream and across* the bow shock.

- Instrument measures $f(\mathbf{v})$ at the spacecraft but not through the ***unmeasured flux tube***. So, we work with ***normalized H-function***,

$$h = \boldsymbol{\Sigma} p_i \log p_i$$

where

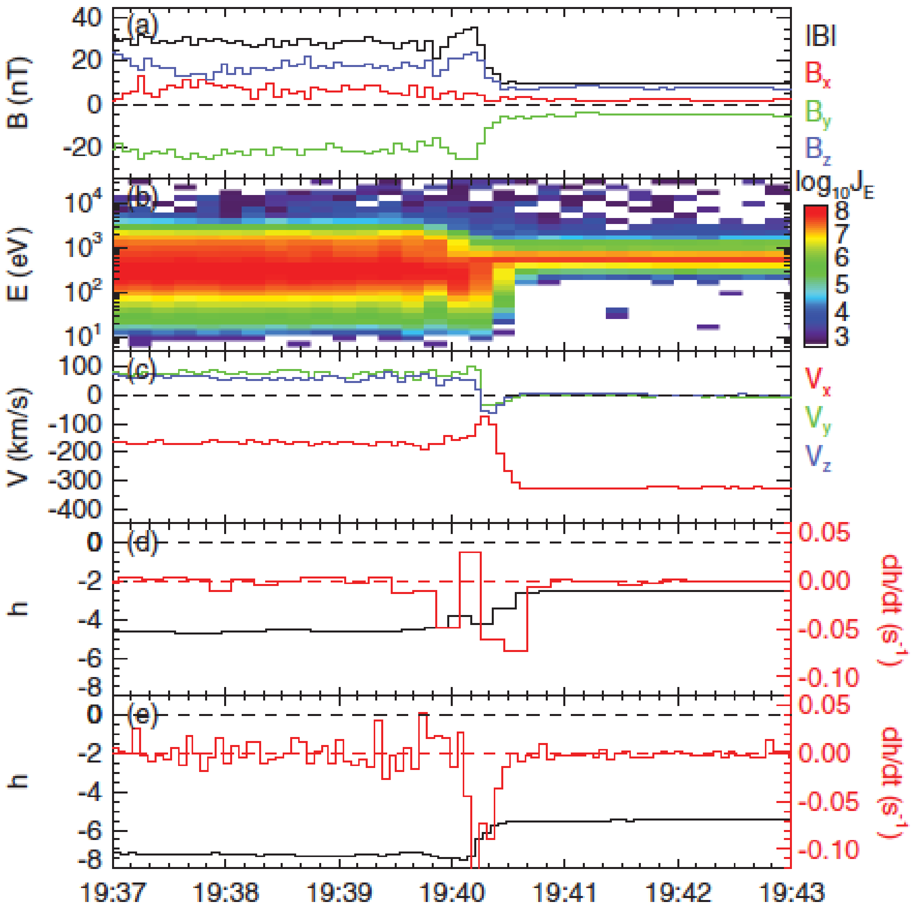
$$p_i = f_i \, \Delta^3 v_i / n$$

n = number density, i indexes sampled phase space volume.

- Note that h is proportional to entropy/particle (entropy density) at SC.

$$\Delta h/\Delta t = [h(t) - h(t - \boldsymbol{\Delta} t)]/\boldsymbol{\Delta} t$$

We calculate $\Delta h/\Delta t$ from successive measurements, where Δt = spin period.



SC outbound, crossed shock at ~1942 UT. Vx ~-320 km/s slowed to -75 km/s and deviated in y and z directions just before crossing the shock. MS plasma flow speed -150 km/s.

Shock parameters:

- $M_A = (V/V_A) \sim 3.0\text{-}3.5$
- $\theta_{BN} \sim 82\text{-}88^o$
- Shock speed along normal 9 km/s.
- Supercritical perpendicular shock.

Ions (Black)

$s = -k_B h$

$\Delta s = \sim 2.9\times10^{-16}$ ergs $^oK^{-1}$

$\Delta s/\Delta t = 0.1\times10^{-16}$ ergs $^oK^{-1}s^{-1}$

Electrons (Red)

$\Delta s = \sim 2.3\times10^{-16}$ ergs $^oK^{-1}$

$\Delta s/\Delta t = 0.18\times10^{-16}$ ergs $^oK^{-1}$

Note: *dh/d*t ~0 in the SW. *dh/d*t changes across shock. In Boltzmann theory *dh/dt* due to collision. Here we sometimes see *dh/dt* >0, not predicted by Boltzmann theory. Significance not understood.

Interpretation

- *Change of entropy Δs ~ **few 10^{-16} ergs $^oK^{-1}$** /particle* for electrons and ions

- These values are of the same order of entropy change as in isolated ***free expansion of ideal gas*** when the volume changes by 2 ($\Delta S = 0.95 \times 10^{-16}$ *ergs* $^oK^{-1}$).

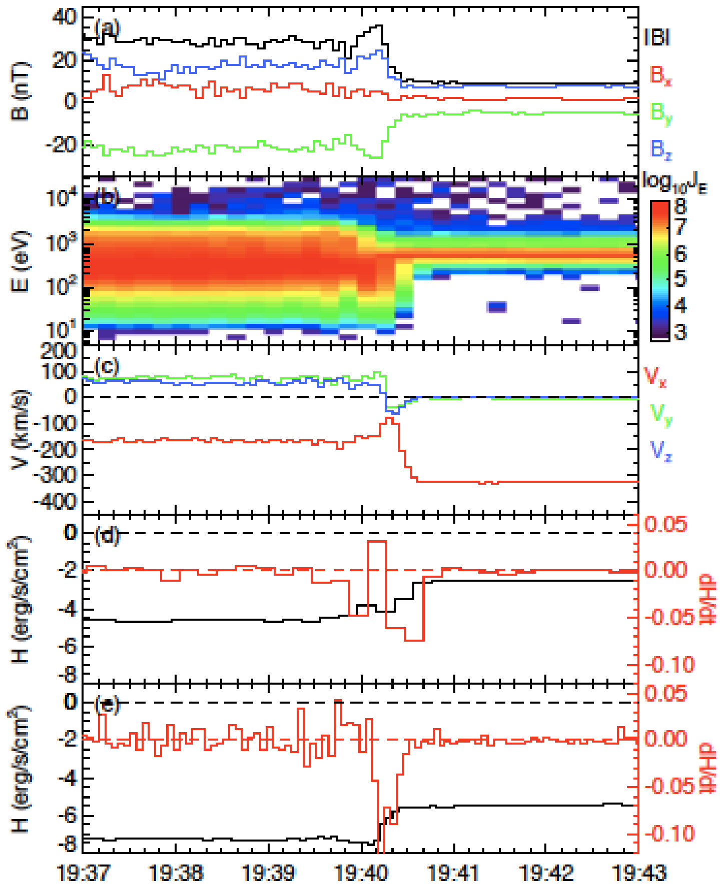
- Also similar to ***ice*** at 0 ^oK ***melting to water*** at same Temperature ($\Delta S \sim 3.3 \times 10^{-16}$ *ergs* $^oK^{-1}$)

- ***Entropy results here all similar*** because energy per particle has an order of magnitude kT. If the *state change* involves an amount of energy corresponding roughly to the original energy, the associated ΔS will be of the order of Boltzmann's constant.

- What this is saying is that the ***compression ratio*** of Earth's shock is ***small*** (typically 2-4)

- ***Astrophysical shocks,*** large compression ratios ($\sim 10^3$), ***expect large ΔS!***

- Multiple distributions at foot and peak: SW, reflected and gyrating distributions.
- Note SW beam not thermalized.

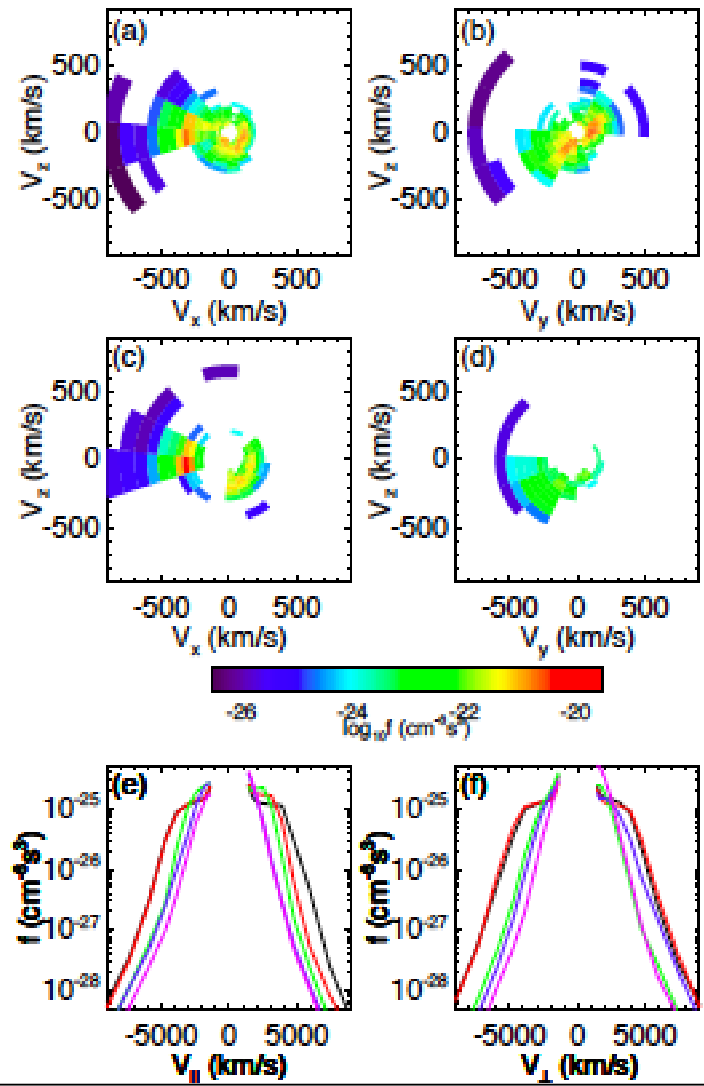


Ions: Peak: SW + diffuse toward Sun

Foot: Gyrating + SW

Electron: SW ~ Maxwellian,

MS = flat-top



Peak

foot

electron

Vlasov Model of Entropy Flux

Multiply Vlasov equation $\frac{\partial}{\partial t} f + v \cdot \nabla f + a \cdot \nabla_v f = 0$ by *log f.*

$$(\log f)\frac{\partial}{\partial t} f + (\log f) v \cdot \nabla f + (\log f) a \cdot \nabla_v f) = 0$$

Use the derivative of a product rule, rewrite and obtain

$$\frac{\partial}{\partial t}(f \log f) + \nabla \cdot (v \log f) + a \cdot \nabla_v (f \log f) - (\partial f / \partial t + v \cdot \nabla f + a \cdot \nabla_v f) = 0$$

Change variables: Let $\boldsymbol{v} = <\boldsymbol{v}> + \boldsymbol{c}$ and $f'(\boldsymbol{c}) = f(<\boldsymbol{v}> + \boldsymbol{c})$;
Integrate over velocity space, obtain

$$\boxed{\frac{\partial}{\partial t} ns + \nabla \cdot (ns <v>) - k_B \nabla \cdot \int (c f' \log f') d^3c = 0}$$

where

$$F = k_B \int (c f' \log f') d^3c$$

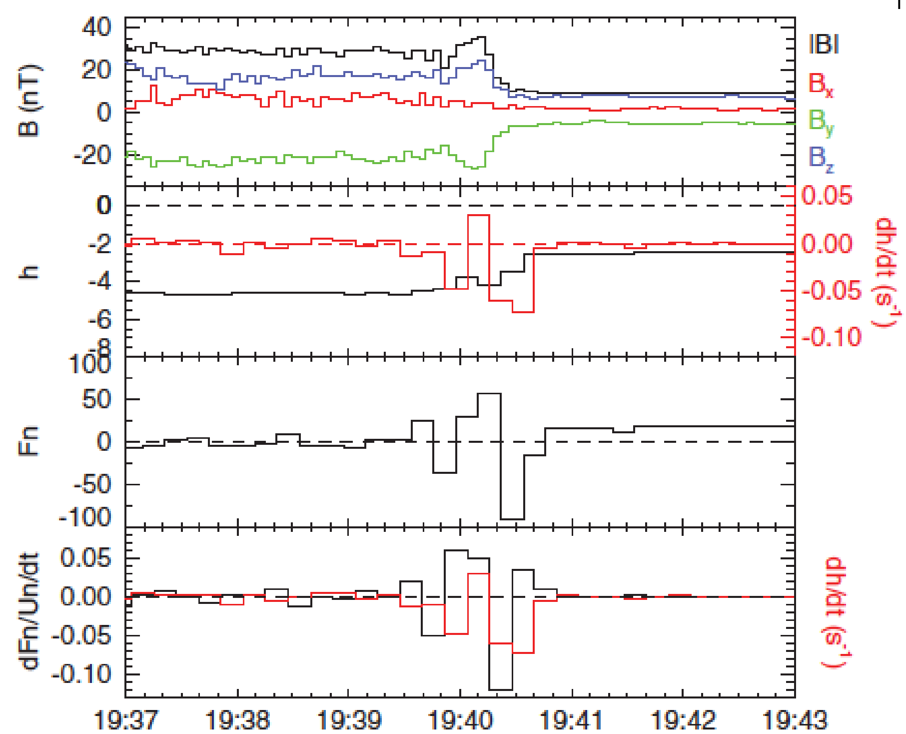
Entropy flux (cont’d)

Assume now steady state, 1D

Entropy flux equation reduces to

$$s_2 - s_1 = \frac{F_{2x} - F_{1x}}{U_1 n_1}$$

The right side gives entropy/particle for non-Mawellian distributions predicted by the simple Vlasov model.



Boltzmann theory assumes

• *system is closed*.

(*Bow shock may not be closed. Measure f along SC track. We never look at the same f a second time*)

• Distribution function *f is homogeneous*

(For bow shock, there is *possible r dependence*)

• *H*-function over a *fixed volume*

(Volume is not measured. *H-function calculated for per particle*)

• *∂p/∂t* due to *collisions* that change internal energy of the system

(Space plasmas collisionless; Don't know what causes ∂p/∂t)

- Developed kinetic model of entropy using Vlasov theory.

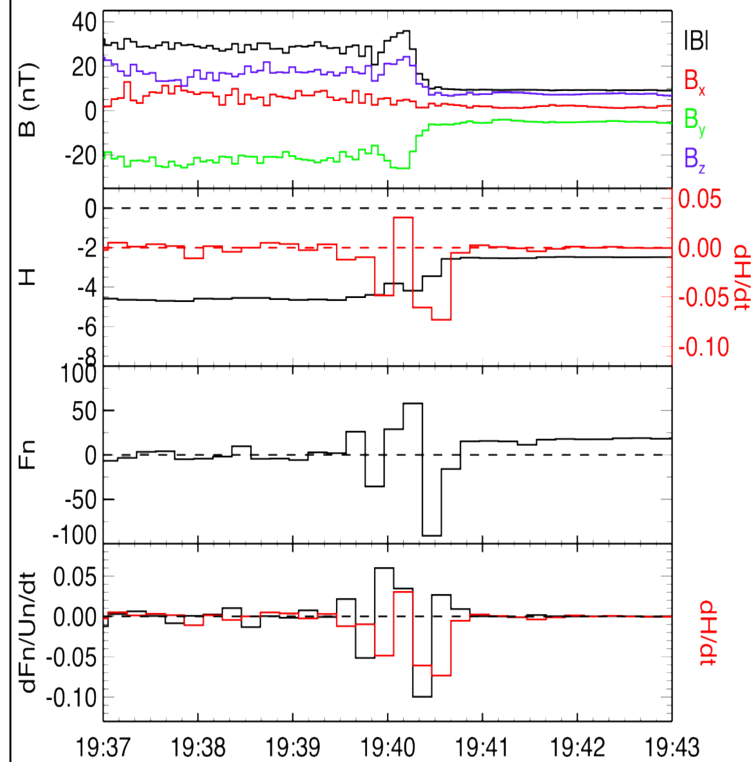
- Derived a kinetic ***entropy flux conservation model*** (under some assumptions)

- This model shows per particle entropy can be generated when the plasma distribution is *non-Maxwellian*, consistent with observations.

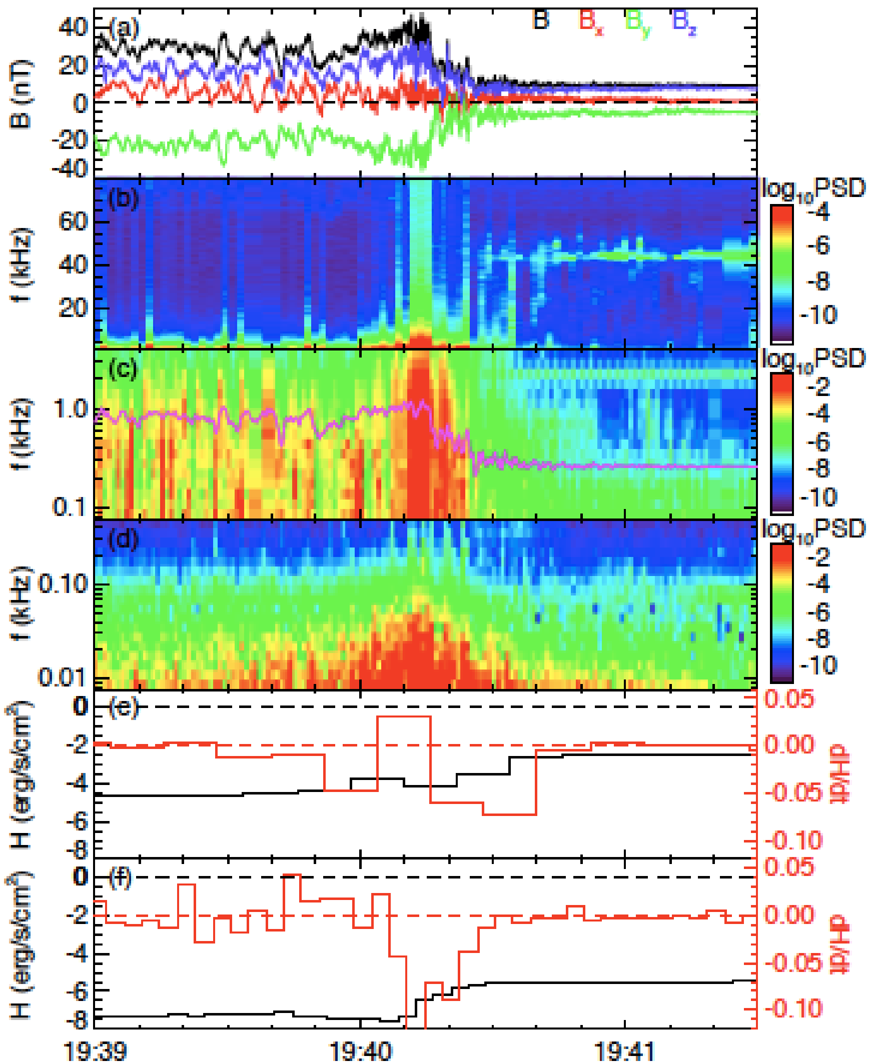
- ***Kinetic Entropy flux conservation equation is satisfied*** is not proof of Vlasov theory but rather gives ***strong support*** **that our analysis generally supports** plasma model of Vlasov theory.

- However, the ***total entropy*** $S = -k_B H$ must vanish for Vlasov plasma.

- The bow shock is somehow shifting the entropy around, increasing ***per particle entropy*** locally. Must ***compensate by a decrease*** of entropy somewhere else.



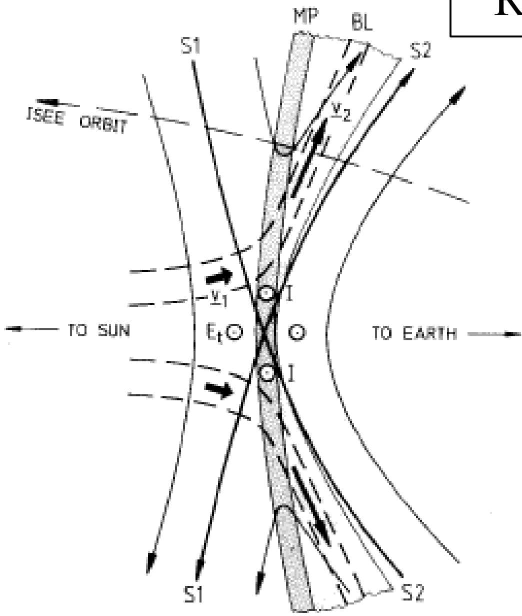
Parks et al., PRL 108, 061102, 2012



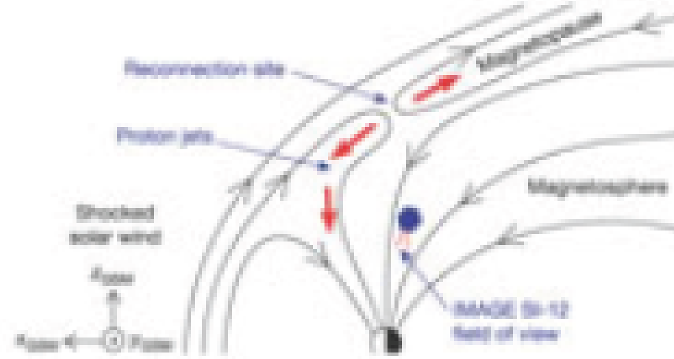
- Analysis included only *f(**v**) of particles*. However, complex EM and ES waves permeate the shock region and entropy generation theory must include the fields.

- No self-consistent theory of entropy that includes particles and waves

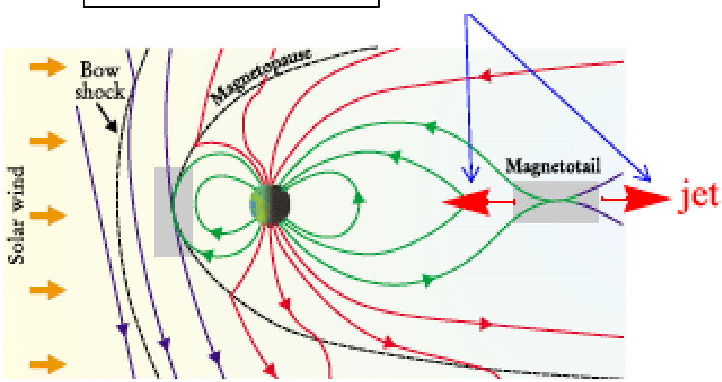
Reconnection concept used in space, solar and astrophysics



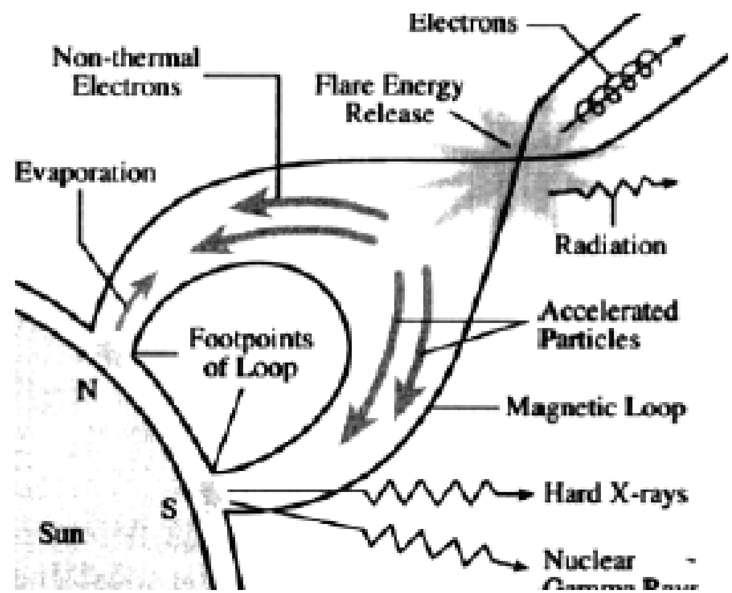
Sonnerup, 1981



Frey, 2003



Paschmann, 2013



ESA

- The original concept of *magnetic field reconnection* in space grew out of the work of *Ronald Giovanelli* (1946) who was interested to learn how *electrons are accelerated* in solar flares.

- He noted that oppositely directed magnetic fields of sunspots create *magnetic neutral points* and that evolving fields would *induce electric fields* that can accelerate particles.

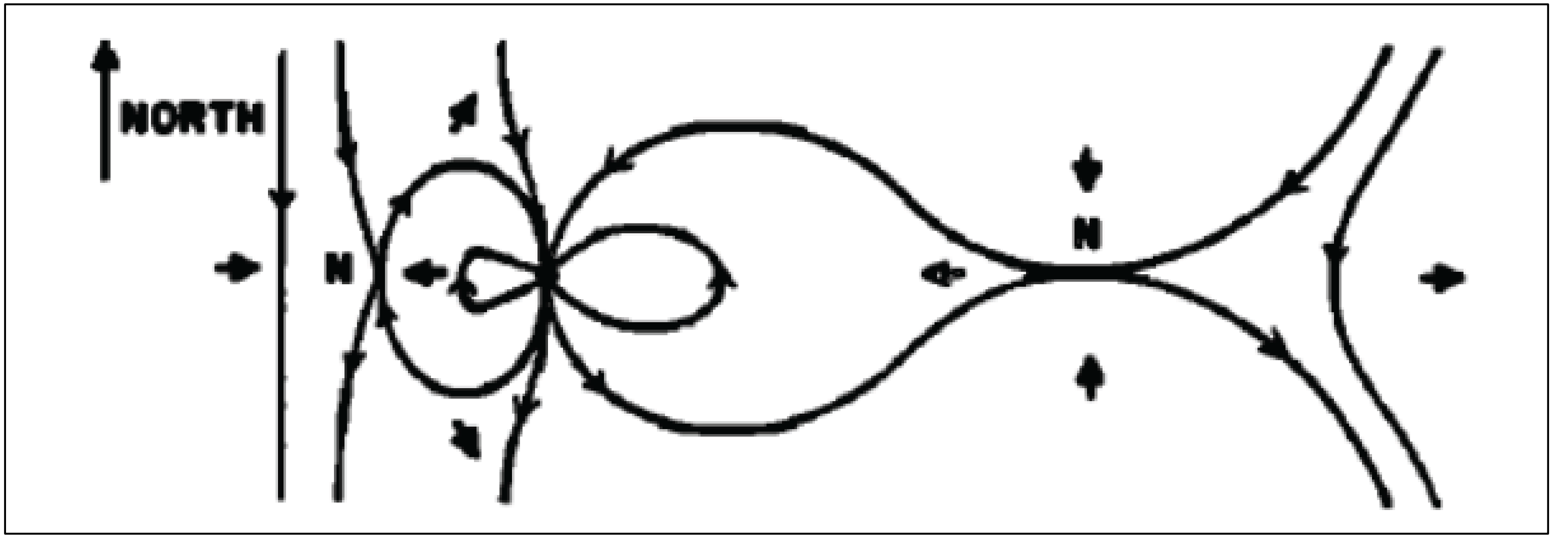
- *F. Hoyle* thought that auroral particles could also be accelerated in the same way and asked *J. Dungey*, his PhD student, to develop Giovanellis' ideas about *particle acceleration in magnetic neutral points* and to apply the theory to auroras.

- Soon after the interplanetary magnetic field (IMF) was discovered, Dungey (1961) realized the presence of *IMF would affect the dynamics of solar wind (SW) interaction with the geomagnetic field* of the static model proposed by Sydney Chapman.

- Dungey noted that the superposition of southward IMF with the geomagnetic field *creates two magnetic neutral points* and the SW flowing in the presence of a neutral point could drive the ionospheric current system.

- He also noted that *flows near the neutral point* were controlled by a *strong current density* existing there.

- He predicted magnetic fields near neutral points are *unstable and would constricted itself* to produce narrow current sheets.

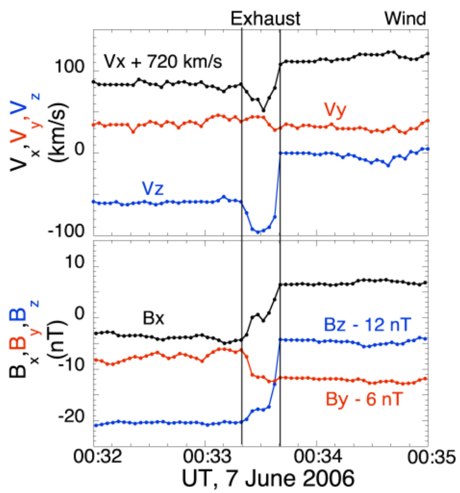


- Dungey is credited with the term *magnetic reconnection*, but he *did not use* the term ``magnetic reconnection" in any of his early papers (Dungey, 1953; 1961).

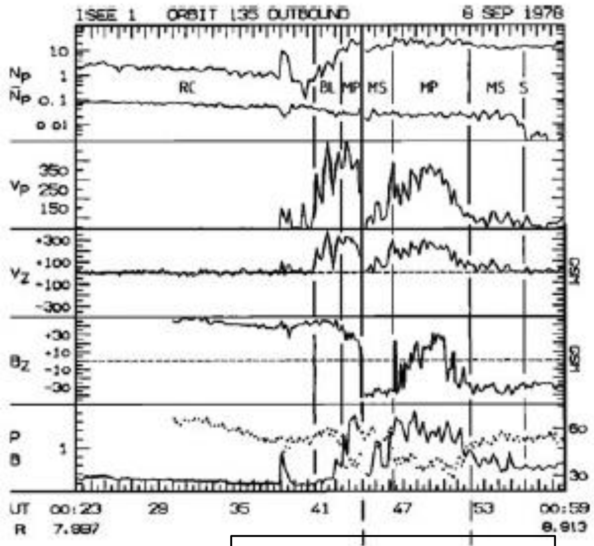
- He cautioned the readers, "*the use of lines of force is a mathematical device and that they are not physical objects; the motion of lines of force is a further device*,.."

- Dungey is *warning us* that *Magnetic charges* from which lines of **B** might emerge (like electric fields) *do not exist*.

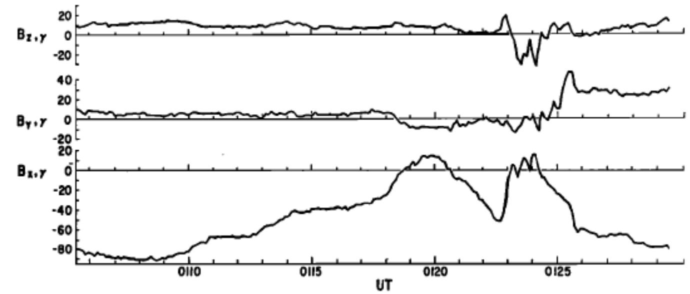
- Since $\nabla \cdot \mathbf{B} = 0$, ***B** lines do not begin or end* and they must *close back on themselves*. True not only for static fields but also for dynamic fields.



Gosling, 2010

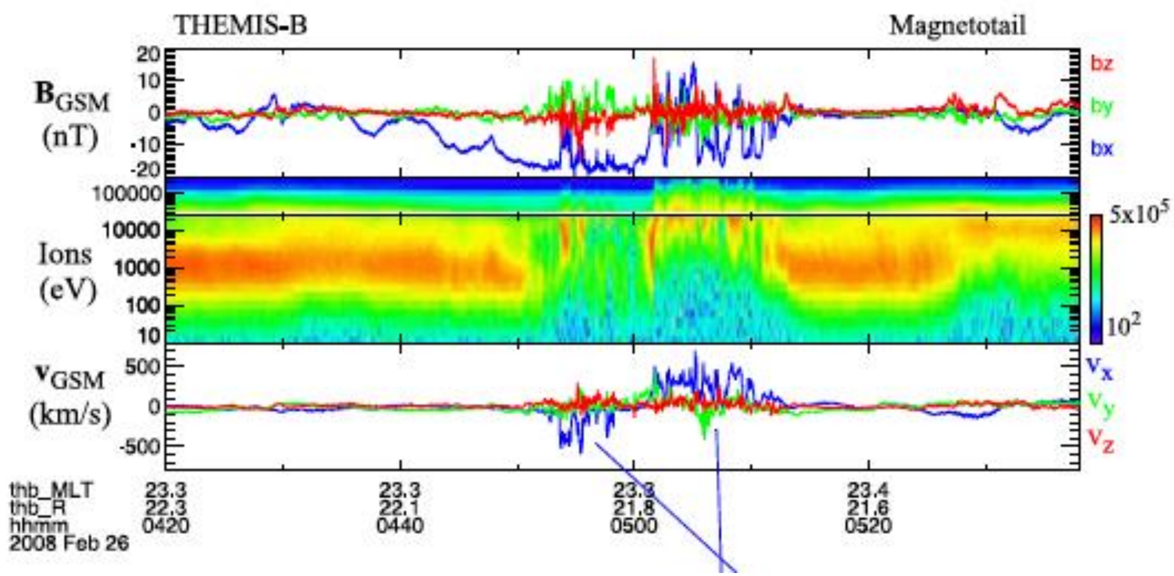


Paschmann, 1981



Laird, 1968

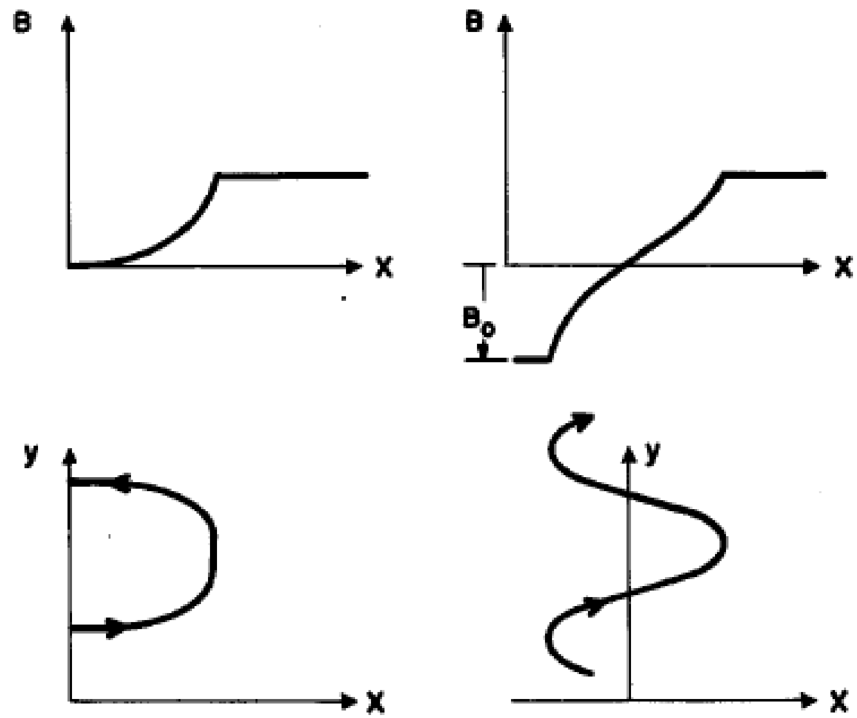
• Observations consistent with reconnection models include *high speed flows* and *magnetic field reversals* at boundaries.



Angelopoulos, 2008; Lui 2009

• Magnetic reconnection physics related to work done earlier.

• Fusion researchers interested in the physics of sheath formation and particle orbits encountering different types of boundary were studied (Mjolness et al., 1961).



(b) (c)

- A boundary can be viewed as a current layer that separates two different regions of plasmas.

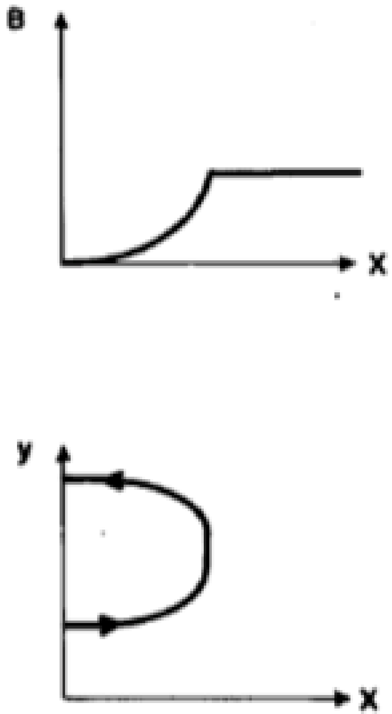
- For example, the dayside magnetopuase separates the magnetosheath and magnetospheric plasmas.

- The magnetic field on the two sides can be pointing in the same or opposite directions.

- Analytical models usually assume a stationary boundary.

- Currents in B-fields in opposite directions are much more complicated because of the presence of *field-free* regions.

- The most commonly used field-reversal geometry is the Harris boundary model.

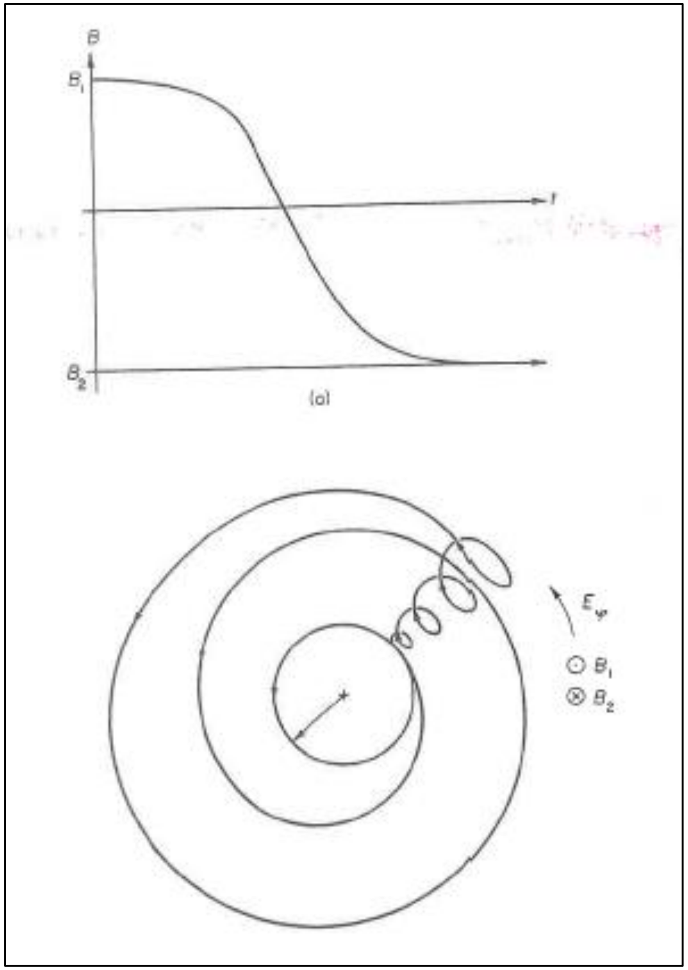


(b)

• Harris solution for the current at the boundary when B on both sides in the same direction (Harris, Nuovo Cimento, Volume XXIII, 1962); see also Longmire (1962) and Parks (2004, Chapter 8)

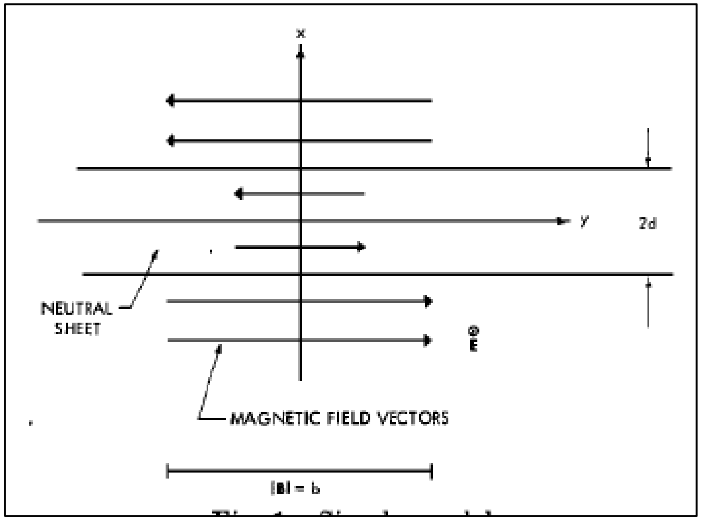
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$J_y = -\frac{n_o e^2}{m_e} \frac{A(x)}{\sqrt{1 - (m_e/m_i)(eA/m_e v_y)^2}}$$



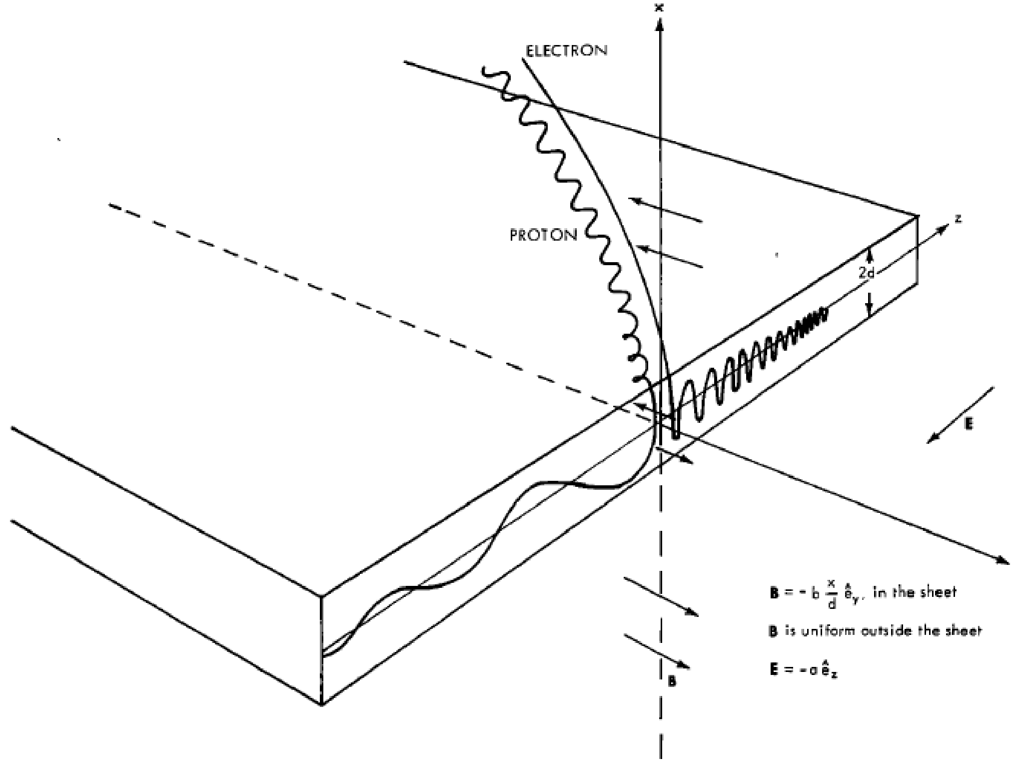
Schmidt, 1962

- A particle which goes through zero magnetic field is *nonadiabatic*.
- As the particle approaches the field reversal region, the *gyroradius becomes larger and larger*, drifting radially outward roughly with $\boldsymbol{E}x\boldsymbol{B}/B^2$ drift.
- The first invariant $\mu \rightarrow \infty$ is not conserved.
- When the particle goes through zero, there is an inflection point in the trajectory and the *radius of curvature* changes sign.
- Even though magnetic field changes sign, the *induced electric field* does not and the particle picks up energy continuously from the electric field after the magnetic field reversal.



$$\mathbf{B} = -b(x/d)\hat{e}_y$$

$$\mathbf{E} = -a\hat{e}_z$$



Speiser (1965) was first to calculate particle orbits in the current sheet of Earth's geomagnetic tail.

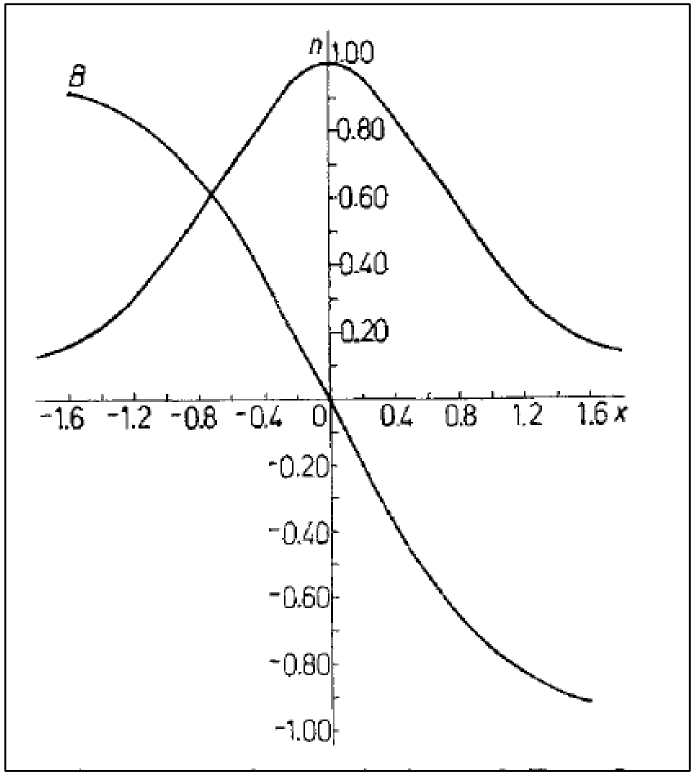
B field is in *opposite* directions. Self-consistent analytical Harris Solution

$$A_y = -\frac{2\kappa T}{qV}\log\ \cosh\left(\frac{xV}{\lambda_D}\right)$$

And obtain

$$B = B_o \tanh\left(\frac{Vx}{\lambda_D}\right)$$

$$n = \frac{n_o}{\cosh^2(Vx/\lambda_D)}$$

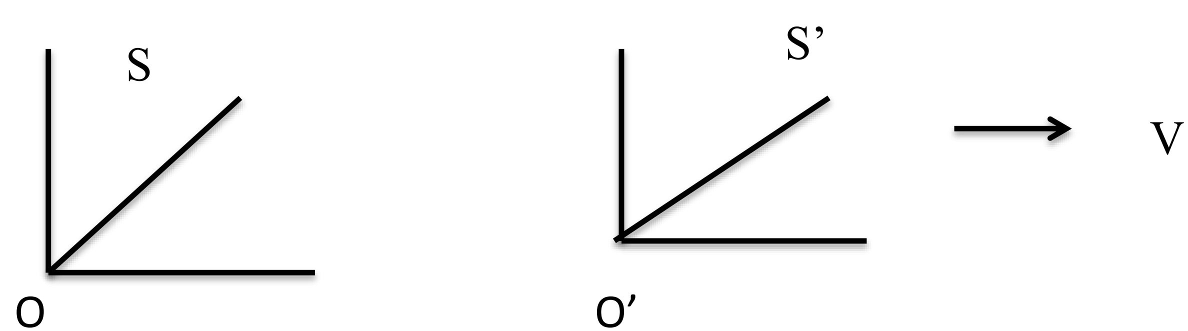


- One needs to perform stablity analysis of the boundary.
- This is being done by PIC simulation at 1D, 2D and 3D (See Drake, 2010).

Lorentz transformation equations for $\mathbf{E}$ and $\mathbf{B}$ can be written vectorially as

$$\begin{aligned} \mathbf{E}'_{\parallel} &= \mathbf{E}_{\parallel} \\ \mathbf{E}'_{\perp} &= \gamma(\mathbf{E} + \mathbf{V} \times \mathbf{B})_{\perp} \\ \mathbf{B}'_{\parallel} &= \mathbf{B}_{\parallel} \\ \mathbf{B}'_{\perp} &= \gamma(\mathbf{B} - \frac{\mathbf{V}}{c^2} \times \mathbf{E})_{\perp} \end{aligned} \qquad (12)$$

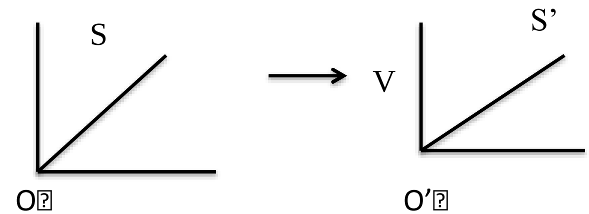
Reminder: || and ⊥ are directions relative to V, velocity of S' frame relative to S-frame.



- We are all familiar with **E** x **B** /B^2 drift of particles for perpendicular **E** and **B** fields. Implicit in this solution is that $|\mathbf{E}| < |\mathbf{B}|$.

- Consider the Lorentz Transformation equations for **E** and **B** fields,

$$\begin{aligned}
\mathbf{E}'_{\|} &= \mathbf{E}_{\|} \\
\mathbf{E}'_{\perp} &= \gamma(\mathbf{E} + \mathbf{V} \times \mathbf{B})_{\perp} \\
\mathbf{B}'_{\|} &= \mathbf{B}_{\|} \\
\mathbf{B}'_{\perp} &= \gamma(\mathbf{B} - \frac{\mathbf{V}}{c^2} \times \mathbf{E})_{\perp}
\end{aligned}$$



- Require $E_{\perp}' = 0$. The second equation shows $(\boldsymbol{E} + \boldsymbol{V}x\boldsymbol{B}) = 0$. So, the velocity of S' frame relative to the S-frame is $\boldsymbol{V} = \boldsymbol{E}\ x\ \boldsymbol{B}/B^2$.

- Now the bottom equation shows $\mathbf{B}_{\perp}' = \gamma(\mathbf{B}-\mathbf{V}\times\mathbf{E}/c^2)_{\perp} \Rightarrow (1- V^2/c^2)^{1/2} = (1 - (E^2/B^2))^{1/2}$, so for the quantity in the square root bracket to be real, $|\mathbf{E}| < |\mathbf{B}|$.

- In the S' frame, there is only magnetic field **B**' and particles are gyrating around **B**'.
- But in the S-frame, the particle is gyrating and drifting with the velocity $\mathbf{V} = \mathbf{E}\ x\ \mathbf{B}/B^2$

• Particle acceleration in *neutral points*.

• At neutral point (sheet), magnetic field vanishes. So we require $|\boldsymbol{B'}| = 0$ in the particle frame S'-frame.

$$\mathbf{E'}_{\perp} = \gamma\,(\mathbf{E}_{\perp} - \mathbf{V} \times \mathbf{B})^{1/2} \qquad (1)$$

$$\mathbf{B'}_{\perp} = \gamma\,[\mathbf{B} - (\mathbf{V}/c^2) \times \mathbf{E}_{\perp}] \qquad (2)$$

(2) Shows that $\quad V = c^2\,(\boldsymbol{E} \times \boldsymbol{B})/E^2 \qquad (3)$

Now $1/\gamma = (1 - V^2/c^2)^{1/2}$: For γ to be real, $(1-[B/E]^2/c^2>1)^{1/2}$. Hence, $|\mathbf{E}| > |\mathbf{B}|$

In the S'-frame, $|\mathbf{B}'| = 0$ and there is only $\mathbf{E'}_{\perp} = \gamma\,\mathbf{E}_{\perp}$.

Since $d\mathbf{p}'/dt = q\,\mathbf{E'}_{\perp}$ particles are continuously accelerated without bounds.

Estimate |**E**| in the neighborhood of neutral point or sheet.

Use Gaussian Unit: 1 Gauss = 1 Stat Volt/cm.
1 Gauss = 10^{-4} T
1 Stat Volt = 300 Volts

Assume Near Neutral sheet, *$B = 10^{-10}$ T*

10^{-6} Gauss = 10^{-6} Stat Volts/cm
= 3x 10^{-4} Volts/cm
= 0.3 mV/cm
= *30 mV/m*

This |***E***|-*field* is large but has been observed on Earth.

- Landau and Lifschitz (1951) examined the case when $|\boldsymbol{E}| = |\boldsymbol{B}|$ and showed that the a particle can be accelerated in the **E**x**B** direction. The solution when $\boldsymbol{B}=(0, 0, B)$ and $\boldsymbol{E}=(0, E, 0)$ is

$$2eEt = p_y^2\left(1 + \frac{\mathcal{E}_z^2}{\alpha^2}\right) + \frac{c^2}{3\alpha^2}p_y^3$$

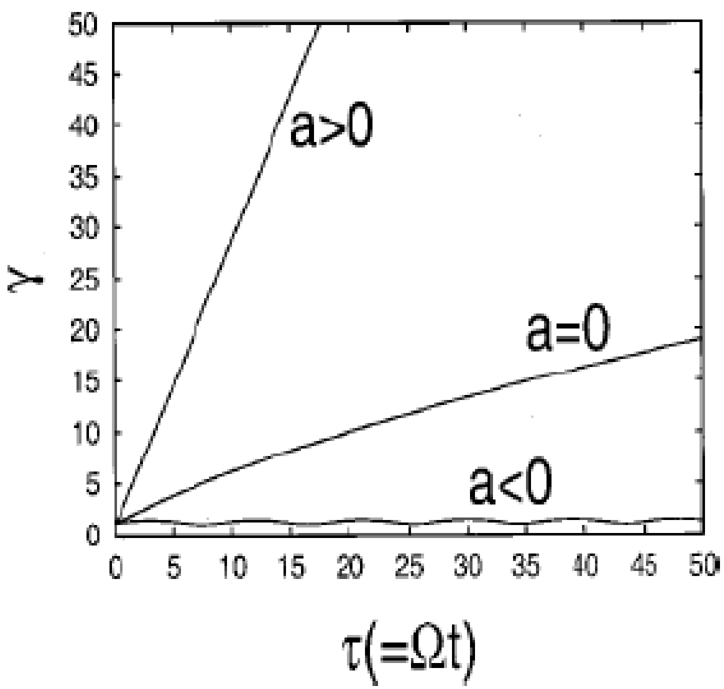
- A cubic equation in p_y and we see that it increases along the electric field direction *(0, E, 0)* without bounds as time t increases.

- Cubic equations were known to the ancient Greeks, Babylonians and Egyptians. In the 7th century, a Chinese mathematician and astronomer Wang Xiatong of Tang Dynasity in a mathematical treatise titled ***Jigu Suanjing*** solved 25 cubic equations of the form $x^3 + qx^2+px+d = 0$.

- Subsequently, *G. Cardan*, an Italian algebraist who lived from 1501-1576 developed an analytical method to solve cubic equations, called Cardan's method.

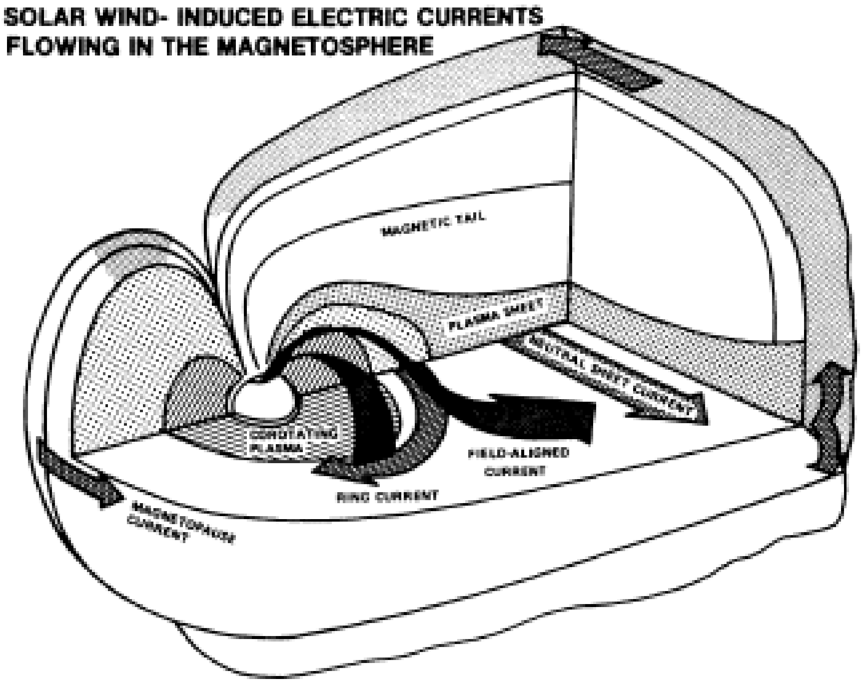
• Takeuchi (2002) solved the Lorentz equation for perpendicular **E** and **B** fields with arbitrary |**E**| and |**B**|.

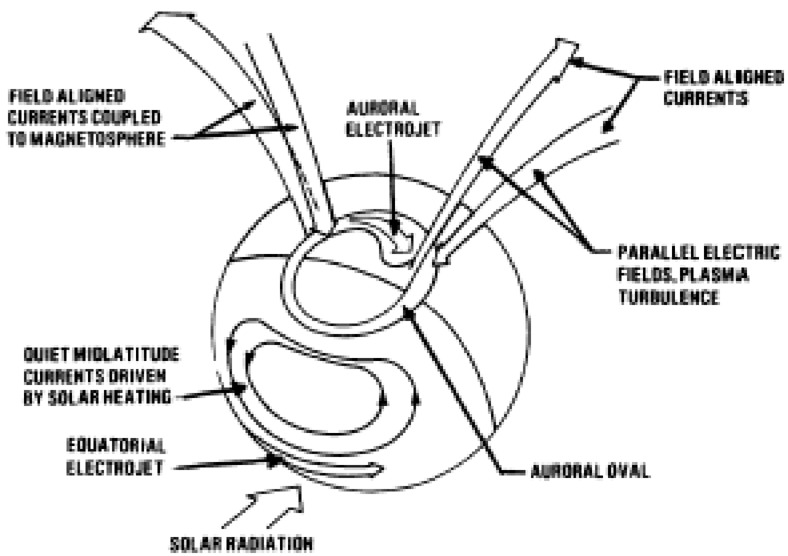
• The solutions show that particles can be accelerated in the **E**x**B** direction.



$a = (E/B)^2 - 1$

From Takeuchida 2002





- Electric field and currents are induced by the solar wind interacting with the geomagnetic field.
- A sketch of what we know about the global magnetospheric and ionospheric current is shown.
- The top sketch shows currents that flow in the magnetosphere and the bottom sketch currents in the ionosphere.
- Description using currents and electric field is self-consistent.
- Electric field and currents measured by instruments can be tested and validated.
- Time dependent picture of Current-Electric field on global scale is still needed to describe dynamic situations.

Recent Review Papers on Magnetic Reconnection:

- Treumann and Baumjohann, Collisionless Magnetic Reconnection in Space Plasmas, *Frontiers in Physics*, 2013.

- Paschmann et al., In-situ observations of reconnection in space, *Space Sci. Rev.*, 2013.

- Lui et al., Critical Issues with Magnetic Reconnection in Space Plasmas, *Space Sci. Rev.*, **116**, 497, 2005.

The End